5. CIRCULAR MOTION

DEFINITION OF CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.

That fixed point is called centre and the distance is called radius of circular path.

KINEMATICS OF CIRCULAR MOTION

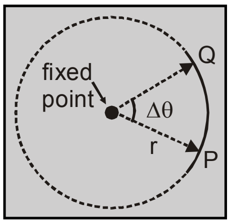
Angular Displacement

Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.

$\Delta\theta$ = angular displacement

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\Delta\theta = \frac{\text{Arc PQ}}{r}$$



GOLDEN KEY POINTS

* Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.
* Its direction is perpendicular to plane of rotation and given by right hand screw rule.
* It is dimensionless and has S.I. unit is "Radian" while other units are degree or revolution.

  2π radian = 360 = 1 revolution

Que. A particle completes 1.5 revolutions in a circular path of radius 2 cm. Find the angular displacement of the particle.

Ans. 3π

Frequency (n) :

Number of revolutions describes by particle per second is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T) :

It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

Angular Velocity (ω) :

It is defined as the rate of change of angular displacement of moving particle .

$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

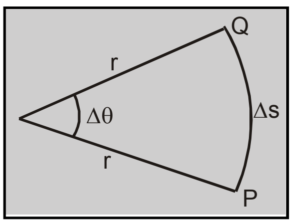
GOLDEN KEY POINTS

* It is an axial vector quantity.
* Its direction is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule.
* Its unit is radian/second.

  Relation between linear and Angular velocity

  $$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \quad \text{or} \quad \frac{\Delta s}{r}$$

  $$\Delta\theta = \frac{\Delta s}{r} \quad \text{or} \quad \Delta s = r\Delta\theta$$



$\therefore \ \dfrac{\Delta s}{\Delta t} = \dfrac{r\Delta\theta}{\Delta t}$ if $\Delta t \to 0$ then $\dfrac{ds}{dt} = r\dfrac{d\theta}{dt}$ $\boxed{v = \omega r}$

$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ (direction of $\vec{v}$ is according to right hand thumb rule)

Average Angular Velocity (ω_{av}) :

$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

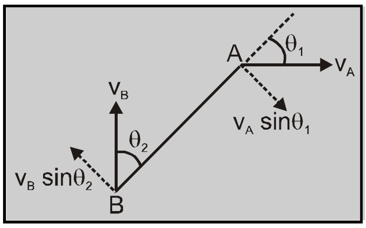
where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity :

The angular velocity at some particular instant $\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$ or $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$

Relative Angular Velocity :

Relative angular velocity of a particle 'A' w.r.t. other moving particle 'B' is the angular velocity of the position vector of 'A' w.r.t. 'B'. That means it is the rate at which position vector of 'A' w.r.t. 'B' rotates at that instant



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

$$= \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\quad \therefore \ \omega_{AB} = \dfrac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

Angular Acceleration (α) :

Rate of change of angular velocity is called angular acceleration.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad \text{or} \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

<u>GOLDEN KEY POINT</u>

* Its an axial vector quantity. It direction is along the axis according to right hand screw rule.
* Unit $\to$ rad/s^2

Ex. A particle revolving in a circular path completes first one third of circumference in 2 s, while next one third in 1 s. Calculate the average angular velocity of particle.

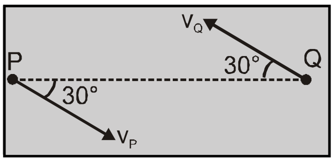
Sol. $\theta_1 = \dfrac{2\pi}{3}$ and $\theta_2 = \dfrac{2\pi}{3}$ total time T = 2 + 1 = 3 s

$$\therefore \ \langle \omega_{av} \rangle = \frac{\theta_1 + \theta_2}{T} = \frac{\frac{2\pi}{3} + \frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9} \ \text{rad/s}$$

Ex. The angular displacement of a particle is given by $\theta = \omega_o t + \dfrac{1}{2}\alpha t^2$, where ω_o and α are constant and $\omega_o = 1$ rad/s, $\alpha = 1.5$ rad/s^2. Find the angular velocity at time t = 2s.

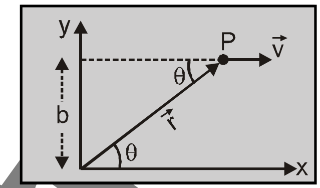
Sol. $\omega = \dfrac{d\theta}{dt} = \omega_0 + \alpha t = 4$ rad/s

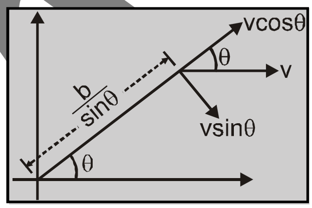
Ex. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°. Calculate the angular velocity of P w.r.t. Q



Sol. $\omega_{PQ} = \dfrac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

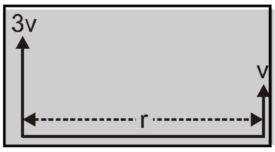
Ex. A particle moving parallel to x-axis as shown in fig. such that at all instant the y-axis component of its position vector is constant and is equal to 'b'. Find the angular velocity of the particle about the origin.





Sol. $\therefore\ \omega_{PO} = \dfrac{v\sin\theta}{\dfrac{b}{\sin\theta}} = \dfrac{v}{b}\sin^2\theta$

Ex. Two points of a rod move with velocity 3v and v seperated by a distance 'r'. Calculate the angular velocity of the rod w.r.t. its end.



Sol. $\therefore\ \omega = \dfrac{3v - v}{r} = \dfrac{2v}{r}$

Ex. The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol. $\alpha = \dfrac{d\omega}{dt} = 1.5 - 6\,t = 0$ or $t = 0.25$ s.

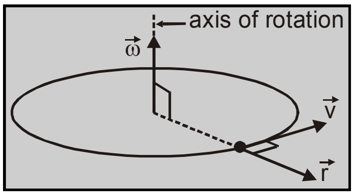
Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol. $\dfrac{d\omega}{dt} = 3t - t^2 \ \Rightarrow\ \int_0^{\omega} d\omega = \int_0^{t}(3t - t^2)dt \ \Rightarrow \omega = \dfrac{3t^2}{2} - \dfrac{t^3}{3} \ \Rightarrow$ at t = 2 s, $\omega = \dfrac{10}{3}$ rad/s

Relation between Angular and Linear Acceleration :

$\vec{v} = \vec{\omega} \times \vec{r}$ ($\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector.)

These three vectors are mutually perpendicular.



but $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\vec{\omega} \times \vec{r}) = \dfrac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \dfrac{d\vec{r}}{dt}$

or $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$ $\left(\dfrac{d\vec{\omega}}{dt} = \vec{\alpha}\right.$ and $\left.\dfrac{d\vec{r}}{dt} = \vec{v}\right)$

or $\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acc. and $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acc.)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are two component of net linear acc.)

Tangential Acceleration :

$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity.

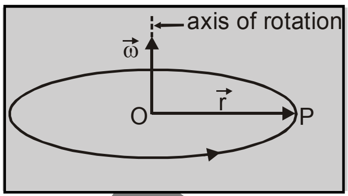
$\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a}_T = \vec{\alpha} \times \vec{r}$

as $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tangential direction.

Magnitude of tangential acceleration, $a_T = \alpha\, r \sin 90 = \alpha r$ ($\vec{\alpha}$ is axial, $\vec{r}$ is radial so that $\vec{\alpha} \perp \vec{r}$)

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration :



$$\vec{a}_C = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r}) \qquad (\because\ \vec{v} = \vec{\omega} \times \vec{r})$$

Let $\vec{r}$ is in $\hat{i}$ direction and $\vec{\omega}$ is in $\hat{j}$ direction

then direction of $\vec{a}_C$ is along $\hat{j} \times (\hat{j} \times \hat{i})$ or $\hat{j} \times (-\hat{k})$ or $-\hat{i}$

opposite direction of $\vec{r}$ i.e., from P to O and it is centripetal direction.

Magnitude of centripetal acceleration, $a_C = \omega v = \dfrac{v^2}{r} = \omega^2 r \Rightarrow \vec{a}_C = \dfrac{v^2}{r}(-\hat{r})$

- * Centripetal acceleration is always perpendicular to the velocity or displacement at each point. So that work done by centripetal force is always zero.
- * When a force acts always perpendicular to the direction of velocity then path described by the particle is circular.

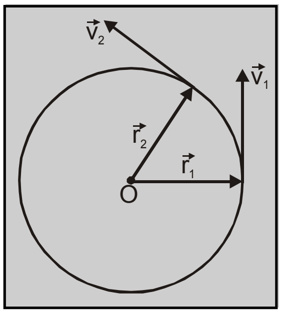
Net Linear Acceleration :

$$\vec{a} = \vec{a}_T + \vec{a}_C \quad \text{and} \quad \vec{a}_T \perp \vec{a}_C \quad \text{so that} \quad |\vec{a}| = \sqrt{a_T^2 + a_C^2}$$

UNIFORM CIRCULAR MOTION

When a particle moves in a circle at a constant speed then the motion is said to be a uniform circular motion.

In such motion, position vector keep changing.



Speed is constant, so that $\vec{a}_T = 0$

Acceleration of particle $\vec{a} = \vec{a}_C = \vec{\omega} \times \vec{v}$

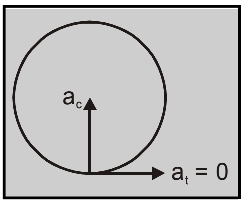
or $a = \omega v$ (but $\omega = \dfrac{v}{r}$) $\quad \therefore\ a = \dfrac{v^2}{r} = \omega^2 r$ = centripetal acceleration

due to centripetal acceleration a result the velocity of the particle keeps on changing the direction i.e. the particle is accelerated.

GOLDEN KEY POINTS

About uniform circular motion :

- * Position vector $(\vec{r})$ is always perpendicular to the velocity vector $(\vec{v})$ i.e. $\vec{r} \cdot \vec{v} = 0$
- * velocity vector is always perpendicular to the acceleration. $\vec{v} \cdot \vec{a} = 0$
- * for circular motion force towards centre (Centripetal force) must act so that direction of $\vec{v}$ keeps on changing which forces the particle to describe a circular path.



- * The work done by centripetal force is always zero.
- * Kinetic Energy = constant
- * $\because\ |\vec{v}|$ = constant so tangential acc. $a_t = 0$ $\quad \therefore\ f_t = 0$
- * Important difference between the projectile motion and uniform circular motion :
  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

Ex. A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then rotated in a horizontal circle making 60 rev/min. Calculate the centripetal acceleration.

Sol. $\omega = 60 \times \frac{2\pi}{60} = 2\pi$ rad/s $\quad \therefore\ a_c = \omega^2 r = 118.4$ m/s^2

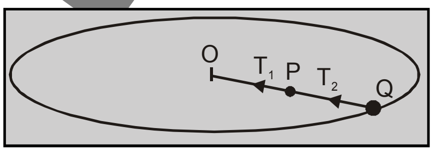
Ex. A stone of mass 0.1 kg tied to one end of a string 1m long is revolved in a horizontal circle at the rate of $\frac{10}{\pi}$ rev/s. Calculate the tension in the string.

Sol. In horizontal circular motion tension $T = m\omega^2 r = (0.10)\left(\frac{10}{\pi} \times 2\pi\right)^2 \times 1 = 40$ N

Hint To Solve Numerical Problems :

(i) Write down the required centripetal force.
(ii) Draw the free body diagram of each component of system.
(iii) Resolve the forces acting on the rotating particle along radius and perpendicular to radius.
(iv) Calculate net radial force acting towards centre of circular path.
(v) Make it equal to required centripetal force.
(vi) For remaining components see according to question.

Ex. Two balls of equal mass are attached to a string at distances 1m and 2m. from one end as shown.The string with the masses is then moved in a horizontal circle with constant speed. What is the ratio of the tension T_1 and T_2?

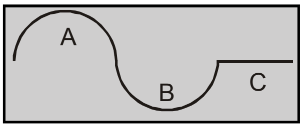


Sol. at $T_1 \xleftarrow{\ P\ } \to T_2$

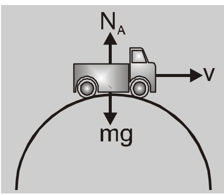
$T_1 - T_2 = m\omega^2$ and $T_2 = 2m\omega^2$

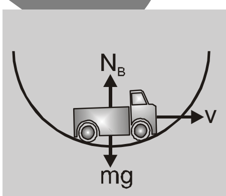
$\therefore\ T_1 = T_2 + m\omega^2 = 3\, m\omega^2 \qquad \therefore\ \frac{T_1}{T_2} = \frac{3}{2}$

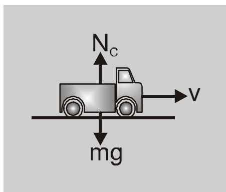
Ex. A car is moving on an uneven road with a uniform speed V. There are three points A, B and C on the road. At which position the reaction of the road maximum?



Sol.







$mg - N_A = \frac{mv^2}{R} \qquad N_B - mg = \frac{mv^2}{R} \qquad N_C = mg$

$\therefore\ N_A = mg - \frac{mv^2}{R} \qquad N_B = mg + \frac{mv^2}{R} \qquad$ hence $N_B > N_C > N_A$

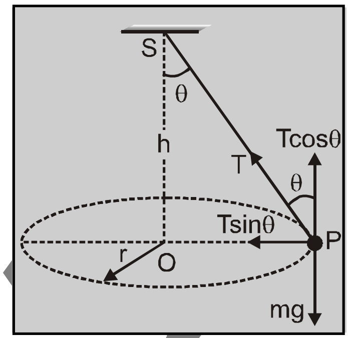
Ex. One meter long string can bear maximum of 0.5 kg mass. A mass of 0.05 kg is tied to one of its end and rotated in a horizontal circle, calculate the max number of revolution so that string does not brakes (rev/min.)

Sol. $m\,\omega^2 r = m_b g = 0.5 \text{ X } 9.8 \therefore\ \omega = \sqrt{98} = 2\pi n \quad \Rightarrow \quad n = 1.576$ rev/s $= 94.5$ rev/min.

MOTION IN HORIZONTAL CIRCLE

Conical Pendulum :

A conical pendulum consits of a body attached to a string of length L, such that it can revolved a horizontal circle with uniform speed. The string traces out a cone in the space.



forces acting on the bob are :

(i) tension in string = T (ii) weight of bob = mg

$$T \sin \theta = \frac{mv^2}{r} \qquad T \cos \theta = mg$$

$$\tan\theta = \frac{v^2}{rg} \qquad \therefore \qquad v = \sqrt{rg\tan\theta}$$

$$\therefore \text{ Time period} = \frac{2\pi r}{v} \qquad \Rightarrow \qquad (\text{Time Period})\ T = \frac{2\pi r}{\sqrt{rg\tan\theta}}$$

$$= 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}} \qquad \left(\because \text{ in } \Delta\ OSP,\ \frac{OP}{SP} = \sin\theta \text{ or } \frac{r}{\sin\theta} = SP = L\right)$$

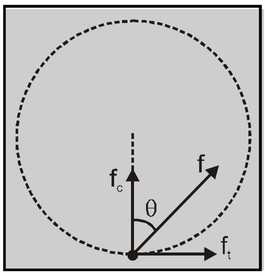
NON- UNIFORM CIRCULAR MOTION

When a particle moving in a circle if the speed of particle increases or decreses then the motion is non-uniform circular motion.

GOLDEN KEY POINTS

* In non- uniform circular motion $|\vec{v}| \neq$ constant $\omega \neq$ constant
* If at any instant v = magnitude of velocity of particle $v = r\omega$
* Tangential acceleration

$$\therefore\ a_T = \frac{dv}{dt} = \text{rate of change of speed } v = \frac{ds}{dt} = \text{speed} \qquad s = \text{arc-length}$$



* Tangential force $f_t = ma_T$
* Centripetal force $f_c = \frac{mv^2}{r} = m\omega^2 r$
* Net acceleration of the particle $\vec{a} = \vec{a}_c + \vec{a}_t \Rightarrow a = \sqrt{a_C^2 + a_T^2}$

  If 'θ ' is the angle made by net force (f) with f_c, then $\tan\theta = \frac{f_t}{f_c}$

  $$\therefore\ \theta = \tan^{-1}\frac{a_t}{a_C}$$

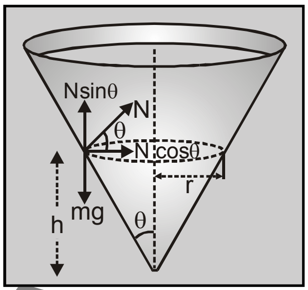
* Net acceleration towards the centre = centripetal acceleration $\Rightarrow a_c = \frac{v^2}{r} = \omega^2 r$

Special Note :

* In both uniform and non- uniform circular motion f_c is perpendicular to velocity. So work done by centripetal force will be zero in both the cases.
* In uniform circular motion $f_t = 0$ as $a_T = 0$

  But in non- uniform circular motion $f_t \neq 0$. Thus there will be work done by tangential force in this case.

Ex. A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9.8 cm. Find the speed of the particle.



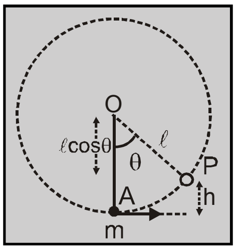
Sol. $N\cos\theta = \dfrac{mv^2}{r}$, $N\sin\theta = mg$

$$\tan\theta = \frac{rg}{v^2} = \frac{r}{h} \quad \text{or } v = \sqrt{hg} = 0.98\,m/s$$

Ex. A car is moving in a circular path of radius 100 m with velocity of 200 m/s such that in each second its velocity increases by 100 m/s. Calculate the net acceleration of the car.

Sol. $a_c = \dfrac{v^2}{r} = 400$, given $a_T = 100\ m/s^2$ $\therefore\ a_{net} = \sqrt{a_c^{\ 2} + a_T^{\ 2}} = 100\sqrt{17}\ m/s^2$

<u>CIRCULAR MOTION IN VERTICAL PLANE</u>



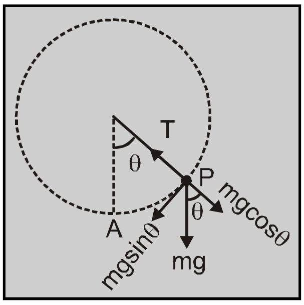
Let a particle of mass m is suspended from a string of length ℓ.

The particle is given a horizontal speed u then it moves in a vertical circle about O. Its velocity V, at point p from h height above the least position. According to energy conservation.

$$0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2 \qquad \text{or } v = \sqrt{u^2 - 2gh}$$

$$\text{or } v = \sqrt{u^2 - 2g\ell\,(1-\cos\theta)} \qquad [\because\ h = \ell(1-\cos\theta)]$$

Tension in string



At point 'p' required centripetal force $= \dfrac{mv^2}{\ell}$

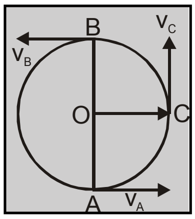
Net force toward centre

$$T - mg\cos\theta = \frac{mv^2}{\ell}$$

$$T = m\left[g\cos\theta + \frac{v^2}{\ell}\right] = \frac{m}{\ell}[u^2 - g\ell(2 - 3\cos\theta)]$$

When string is vertical and particle is at lowest position (A) then tension in the string

$$T_A = \frac{mv_A^2}{\ell} + mg = \frac{mu^2}{\ell} + mg \qquad [\theta = 0°]$$



When particle is at point B of the circle –

$$T_B = \frac{mv_B^2}{\ell} - mg \qquad (\theta = 180\ )$$

$$= \frac{mu^2}{\ell} - 5mg \qquad [\text{by law of conservation of energy } v_B^{\ 2} = u^2 - 2g\,(2\ell)]$$

When particle is at point C of the circle –

$$T_C = \frac{mv_C^2}{\ell} \qquad (\theta = 90\ )$$

$$= \frac{mu^2}{\ell} - 2mg \qquad [\text{by law of conservation of energy } v_C^{\ 2} = u^2 - 2g(\ell)]$$

Thus we can conclude $T_A > T_C > T_B$

$$T_A - T_B = 6mg$$
$$T_A - T_C = 3mg$$
$$T_C - T_B = 3mg$$

Cases :

(a) if $u > \sqrt{5g\ell}$

In this case tension in the string will not be zero at any point, which implies that the particle will continue the circular motion.

(b) $u = \sqrt{5g\ell}$ $\rightarrow$ just completes the loop (at point B tension becomes zero, $T_B = \frac{mu^2}{\ell} - 5mg$)

critical velocity to complete the loop is $u = v_A = \sqrt{5g\ell}$

$v_B = \sqrt{g\ell}$ $[\ v_B^{\ 2} = v_A^{\ 2} - 4g(\ell)\ ]$

$v_c = \sqrt{3g\ell}$ $[\ v_C^{\ 2} = v_A^{\ 2} - 2g(\ell)]$ and $T_A = 6mg$, $T_B = 0$, $T_c = 3mg$

(c) $\sqrt{2g\ell} < u < \sqrt{5g\ell} \Rightarrow$ not complete the loop and leave the circular path.

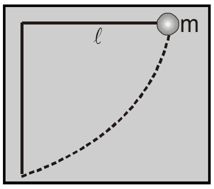
Tension becomes zero between points C and B but speed $v \neq 0$ in this case.

(d) $u = \sqrt{2g\ell}$ $\Rightarrow$ T = 0 and v = 0 at point C.

Particle will horizontal about point A.

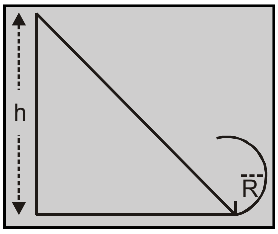
(e) $u < \sqrt{2g\ell}$ v = 0 in between A and C, but T $\neq$ O oscillates about 'A'.

Ex. A particle of mass 'm' tied at with a string of length ℓ is released from horizontal position as shown is fig. Find the velocity at the lowest portion.



Sol. apply COME $mg\ell = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2g\ell}$

Ex. A 4 kg ball swing in a vertical circle at the end of a cord 1m long. Find the maximum speed at which it can swing if the cord can sustain maximum tension of 163.6 N.

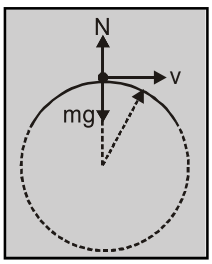


Sol. $T = \frac{mv^2}{r} + mg$ $\therefore$ v = 5.6 m/s

Ex. A ball is released from height 'h' as shown, which of the following condition hold good for the particle to complete the circular path.

Sol. $v = \sqrt{2gh} \geq \sqrt{5gR} \Rightarrow \sqrt{2gh} \geq \sqrt{5gR} \Rightarrow h \geq \frac{5}{2}R$

Ex. A circular overbridge having radius 20m, what is the maximum speed with which a car can cross the bridge without leaving contact with ground at the heighest point (g = 9.8 m/sec^2)

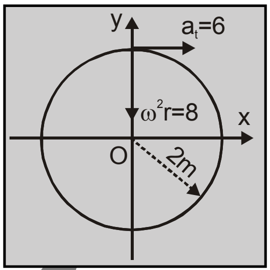


Sol. For motion, $mg - N = \frac{mv^2}{r}$

When reaction N becomes zero, contact is about to leave

$\therefore mg = \frac{mv^2}{r}$ or $v = \sqrt{rg} = 14$ m/s

Ex. A ring rotates about z-axis as shown in fig. The plane of rotation is x-y plane. At a certain instant the acceleration of a particle P (Shown in fig.) on the ring is $(6\hat{i} - 8\hat{j})$ m/sec^2. At that instant what is the angular acceleration and angular velocity of the ring ?

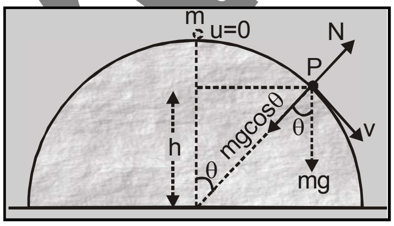


Sol. $\vec{a} = \vec{a}_T + \vec{a}_C$ here $\vec{a}_C$ is along $-\hat{j}$ and $\vec{a}_T$ is along $\hat{i}$

given $\vec{a} = 6\hat{i} - 8\hat{j}$ $\Rightarrow a_T = 6 = \alpha\, r$ and $a_C = 8 = \omega^2 r$

now $\vec{\alpha} = \frac{6}{2} = 3$ rad/s$^2(-\tilde{k})$, $\vec{\omega} = 2$ rad/s$(-\tilde{k})$

Ex. A particle of mass 'm' slide down form the vertex of hemisphere, without any intial velocity. At what height from the horizontal will the particle leave the sphere.



Sol. At point P $\quad mg\cos\theta - N = \frac{mv^2}{R}$

To leave the contact N = 0 so $v^2 = gR\cos\theta$

by law of conservation of energy $0 + mg\,R = \frac{1}{2}mv^2 + mgh$

$v^2 = 2g(R - h) = gR\cos\theta \quad (\because \cos\theta = \frac{h}{R}) \Rightarrow h = \frac{2}{3}R$

CIRCULAR TURNING AND BANKING OF ROADS

External forces acting on car

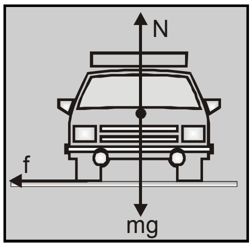
(I) Weight 'mg'

(ii) Normal contact force N

(iii) Friction force = f

f $\rightarrow$ static friction force and it is self adjustable.

The tyres get a tendency to skid outwords and frictional force which opposes this skidding acts. towards the centre.



for a safe turn $f > \frac{mv^2}{r}$

There is a limit to magnitude of the frictional force.

$f = \mu N \quad \mu$ = coefficient of friction

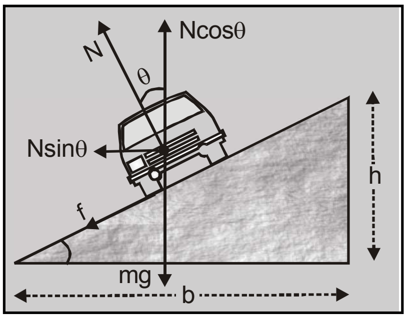
[N = mg for vertical equilibrium]

or $\frac{mv^2}{r} \le \mu mg \quad \Rightarrow \mu \ge \frac{v^2}{rg} \Rightarrow v_{max} = \sqrt{\mu rg}$

friction is not always reliable at circular turns if high speed and sharp turns are involved.

To avoid the dependence on friction, the roads are banked at the turn. So that the outer part of the road is some what lifted up as compared to the inner part. This is known as banking of the road.

At correct speed, horizontal component of 'N" is sufficient to produce the acceleration towards the centre and the self adjustable frictional force keeps its value zero.



$N\sin\theta = \frac{mv^2}{r}$ ...(1)

$N\cos\theta = mg$ ...(2)

$\tan\theta = \frac{v^2}{rg} = \frac{h}{b}$

here h is height of the outer edge and b is width of the road.

Ex. When the string of a conical pendulum makes an angle of 45 with the vertical, its time period is T_1. When the string makes an angle of 60 with the vertical, time period is T_2. Then find the value of $\frac{T_1^2}{T_2^2}$.

Sol. Time period of conical pendulum is $T = 2\pi\sqrt{h/g}$ where $h = \ell\cos\theta$

Therefore $T \propto \sqrt{\cos\theta} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{\cos 45°}{\cos 60°} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

Ex. A particle moves in a circle of radius 20 cm. Its linear speed is given by $v = (3t^2 + 5t)$ where t is in seconds and v is in m/s. Find the resultant acceleration at t = 1 s.

Sol. Tangential acceleration $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 5t) = 6t + 5$

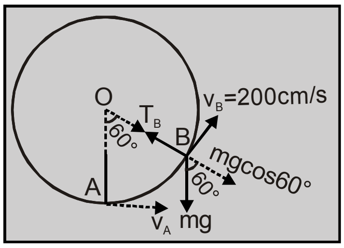
at t = 1 s : $a_t = 6 \times 1 + 5 = 11$ m/s.^{2} and velocity $v = 3t^2 + 5t = 3 \times 1 + 5 \times 1 = 8$ m/s

$\therefore$ radial acc $a_r = \frac{v^2}{r} = \frac{(8)^2}{0.2} = \frac{64}{0.2} = 320$ m/s^2

$\therefore$ Resultant acceleration $a = \sqrt{(a_t)^2 + (a_r)^2} = \sqrt{(11)^2 + (320)^2} = 320.189$ m/s^2

Ex. A particle of mass 100 gm is suspended from the end of a weightless string of length 100cm. and is allowed to swing in a vertical plane. The speed of the mass is 200 cm/s. when the string makes an angle of 60 with the vertical axis. Determine the tension in the string at 60 .

Sol. Tension at point (B)

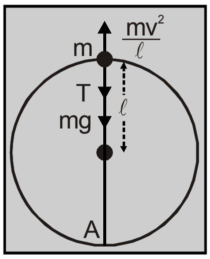


$F_{\text{net towards O}} = F_{cp} \Rightarrow T_B - mg\cos 60 = \frac{mv_B^2}{\ell}$

$T_B = \frac{mv_B^2}{\ell} + mg\cos 60 = \frac{100(200)^2}{100} + 100 \times (1000) \times \frac{1}{2}$

$= 40,000 + 50,000 = 90,000$ dyne $= 0.9 \times 10^5$ dyne $= 0.9$ N

Ex. In a vertical circular motion, tension at the highest point is equal to the weight of the particle, then find the speed and tension at the lowest point. Mass of the particle is m = 9.5kg and length of string is $\ell = 10$ m ($g = 10$ m/s^2)

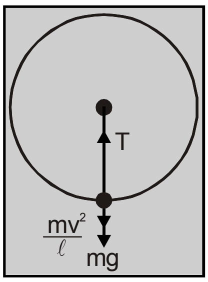


Sol. At the highest point $T + mg = \frac{mv^2}{\ell}$ given T = mg

$\therefore$ $mg + mg = \frac{mv^2}{\ell} \Rightarrow v^2 = 2\ell g \Rightarrow v = \sqrt{2\ell g}$

by conservation of mechanical energy between top most & lowest point

$\frac{1}{2}mv^2 + mg(2\ell) = \frac{1}{2}mv_1^2 \Rightarrow \frac{1}{2}m(2\ell g) + 2mg\ell = \frac{1}{2}mv_1^2$



$6g\ell = v_1^2 \Rightarrow v_1 = \sqrt{6g\ell}$

At lowest point $T = mg + \frac{mv_1^2}{\ell} = mg + \frac{m}{\ell} \times 6g\ell$

$T = 7mg = 665$N and $v_1 = \sqrt{6g\ell} = 24.49$ m/s

STD. XII

TIME :

Prof. SAMEER UNIA'S PHYSICS TUTORIALS

DATE :

PHYSICS

TOPIC : CIRCULAR MOTION - TUTORIAL SHEET - I

STUDENT NAME : ____________________

1. A body of mas 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolution/sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be
(1) 1.88 m/s, 35.5 m/s^2, 35.5 N
(2) 2.88 m/s, 45.5 m/s^2, 45.5 N
(3) 3.88 m/s, 55.5 m/s^2, 55.5 N
(4) None of these

2. The force required to keep a body in uniform circular motion is :- [AFMC 2003]
(1) Centripetal force (2) Centrifugal force (3) Resistance (4) None of the above

3. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is
(1) $\frac{ML\omega^2}{2}$ (2) $\frac{ML^2\omega}{2}$ (3) $ML\omega^2$ (4) $\frac{ML^2\omega^2}{2}$

4. A sphere of mass m is tied to end of string of length ℓ and rotated through the other end along a horizontal circular path with speed v. The work done in full horizontal circle is
(1) 0 (2) $\left(\frac{mv^2}{\ell}\right).2\pi\ell$ (3) $mg.2\pi\ell$ (4) $\left(\frac{mv^2}{\ell}\right).\ell$

5. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration
(1) $r\omega^2$ (2) Constant (3) Zero (4) None of the above

6. A particle of mass (m) revolving in horizontal circle of radius (R) with uniform speed v. When particle goes from one end to other end of diameter, then

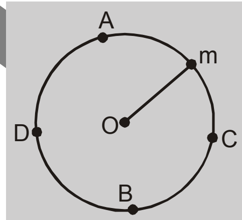
(1) K.E. changes by $\frac{1}{2}mv^2$ (2) K.E. change by mv^2

(3) no change in momentum (4) change in momentum is 2 mv

7. A mass is performing vertical circular motion (see figure). If The average velocity of the particle is increased, then at which point maximum breaking possibility of the string

(1) A (2) B

(3) C (4) D



8. The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5 m, then linear velocity of the wheel is

(1) 70 m/s (2) 35 m/s (3) 30 m/s (4) 20 m/s

9. The angular velocity of a particle rotating in a circular orbit 100 times per minute is

(1) 1.66 rad / s (2) 10.47 rad / s (3) 10.47 degree / s (4) 60 degree / s

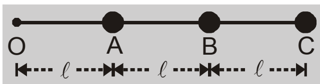
10. A motor cycle driver doubles its velocity when he is taking a turn. The force exerted outwards will become

(1) double (2) half (3) 4 times (4) $\frac{1}{4}$ times

11. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is

(1) 3 : 5 : 7 (2) 3 : 4 : 5

(3) 7 : 11 : 6 (4) 3 : 5 : 6



12. Two particles having mass 'M' and 'm' are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$ (3) 1 (4) $\sqrt{\frac{R}{r}}$

13. A stone is tied to one end of string 50 cm long and is whirled in a horizontal circle with constant speed. If the stone makes 10 revolutions in 20 s, then what is the magnitude of acceleration of the stone
(1) 493 cm/s^2 (2) 720 cm/s^2 (3) 860 cm/s^2 (4) 990 cm/s^2

14. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
(1) 250 N (2) 1000N (3) 750N (4) 1200 N

15. A stone of mass 0.2 kg is tied to one end of a thread of length 0.1 m whirled in a vertical circle. When the stone is at the lowest point of circle, tension in thread is 52N, then velocity of the stone will be
(1) 4 m/s (2) 5 m/s (3) 6 m/s (4) 7 m/s

16. A stone is attached to the end of a string and whirled in horizontal circle, then
(1) its linear and angular momentum are constant (2) only linear momentum is constant
(3) its angular momentum is constant but linear momentum is variable
(4) both are variable

17. Angular velocity of minute hand of a clock is
(1) $\frac{\pi}{30}$ rad/s (2) 8π rad/s (3) $\frac{2\pi}{1800}$ rad/s (4) $\frac{\pi}{1800}$ rad/s

18. A car moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of $2 m/s^2$. The acceleration of the car is
(1) 9.8 m/s^2 (2) 1.8 m/s^2 (3) 2 m/s^2 (4) 2.7 m/s^2

19. If a particle is rotating in a horizontal circle, what will happen?
(1) no force is acting on particle (2) velocity of particle is constant
(3) particle has no acceleration (4) no work is done

20. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s. at the end of the second revolution after motion has begun, the tangential acceleration is
(1) 40 m/s^2 (2) 640 π m/s^2 (3) 160 π m/s^2 (4) 40 π m/s^2

21. For a particle in a non-uniform accelerated circular motion
    (1) velocity is radial and acceleration is transverse only
    (2) velocity is transverse and acceleration is radial only
    (3) velocity is radial and acceleration has both radial and transverse components
    (4) velocity is transverse and acceleration has both radial and transverse components

22. A mass m is attached to the end of a rod of length ℓ . The mass goes around a verticle circular path with the other end hinged at the centre. What should be the minimum velocity of mass at the bottom of the circle so that the mass completes the circle ?
    (1) $\sqrt{4g\ell}$ (2) $\sqrt{3g\ell}$ (3) $\sqrt{5g\ell}$ (4) $\sqrt{g\ell}$

23. A fighter plane is moving in a vertical circle of radius 'r'. Its minimum velocity at the highest point of the circle will be
    (1) $\sqrt{3gr}$ (2) $\sqrt{2gr}$ (3) $\sqrt{gr}$ (4) $\sqrt{\frac{gr}{2}}$

24. A stone is tied to a string of length 'ℓ' and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed 'u'. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is
    (1) $\sqrt{u^2 - g\ell}$ (2) $u - \sqrt{u^2 - 2g\ell}$ (3) $\sqrt{2g\ell}$ (4) $\sqrt{2(u^2 - g\ell)}$

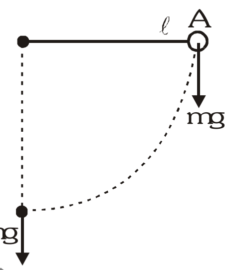
25. A particle is kept at rest at the top of a sphere of diameter 42 m. When disturbed slightly, it slides down. At what height 'h' from the bottom, the particle will leave the sphere
    (1) 14 m (2) 28 m (3) 35 m (4) 7 m

26. A stone tied to the end of a string of 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone
    (1) $\pi^2 ms^{-2}$ and direction along the tangent to the circle.
    (2) $\pi^2 ms^{-2}$ and direction along the radius towards the centre.
    (3) $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre.
    (4) $\pi^2 ms^{-2}$ and direction along the radius away from the centre.

27. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is
(1) 0,0 (2) 0, 10 m/s (3) 10 m/s, 10 m/s (4) 10 m/s, 0

28. A fly wheel rotating at 600 rev/min is brought under uniform decceleration and stopped after 2 minutes, then what is angular decceleration in rad/sec^2 ?
(1) $\frac{\pi}{6}$ (2) $10\ \pi$ (3) $\frac{1}{12}$ (4) 300

29. Mass m is released from point A as shown in figure then tension in the string at the point B will be-
(1) mg (2) 2mg
(3) 3mg (4) 4mg



30. A roller coaster is designed such that riders experiece"weightlessness" as they go round the top of a hill whose radius of curvature is 20m. The speed of the car at the top of the hill is between:-
(1) 16m/s and 17m/s (2) 13m/s and 14 m/s (3) 14m/s and 15m/s (4) 15m/s and 16 m/s

31. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if
(1) $r \geq \frac{\mu g}{\omega^2}$ (2) $r = \mu g \omega^2$ (3) $r < \frac{\omega^2}{\mu g}$ (4) $r \leq \frac{\mu g}{\omega^2}$

32. A particle moves in a circle of radius 5cm with constant speed and time period 0.2 π s. The acceleration of the particle is
(1) 15 m/s^2 (2) 25 m/s^2 (3) 36 m/s^2 (4) 5 m/s^2

33. A car of mass 1000 kg negotiates a banked curve of radius 90m on a fictionless road. If the banking angle is 45 , the speed of the car is
(1) 5ms^{-1} (2) 10ms^{-1} (3) 20ms^{-1} (4) 30ms^{-1}

34. A car of mass m is moving on a level circular track of radius R. If μ_s represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by
(1) $\sqrt{mRg/\mu_s}$ (2) $\sqrt{\mu_s\ Rg}$ (3) $\sqrt{\mu_s\ mRg}$ (4) $\sqrt{Rg/\mu_s}$

STD. XII **DATE :**

TIME :

Prof. SAMEER UNIA'S PHYSICS TUTORIALS

PHYSICS

TOPIC : CIRCULAR MOTION - TUTORIAL SHEET - II

STUDENT NAME : ______________________________

1. A particle of mass 'm' describes a circle of radius (r). The cetripetal acceleration of the particle is $\frac{4}{r^2}$. The momentum of the particle

(1) $\frac{2m}{r}$ (2) $\frac{2m}{\sqrt{r}}$ (3) $\frac{4m}{r}$ (4) $\frac{4m}{\sqrt{r}}$

2. A particle is moving around a circular path with uniform angular speed (ω). The radius of the circular path is (r). The acceleration of the particle is

(1) $\frac{\omega^2}{r}$ (2) $\frac{\omega}{r}$ (3) $v\omega$ (4) vr

3. Let 'θ' denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is (m), then the tension in string is mg cos θ

(1) always (2) never

(3) at the extreme positions (4) at the mean position

4. The angular acceleration of particle moving along a circular path with uniform speed :–

(1) uniform but non zero (2) zero

(3) variable

(4) such as can not be predicted from given information

5. A car moves on a circular road, describing equal angles about the centre in equal intervals of times which of the statements about the velocity of car are true

(1) velocity is constant

(2) magnitude of velocity is constant but the direction of velocity change

(3) both magnitude and direction of velocity change

(4) velocity is directed towards the centre of circle

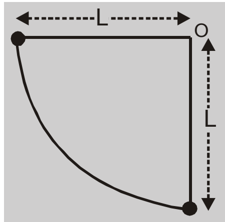
6. A pendulum bob has a speed 3 m/s while passing through its lowest position, length of the pendulum is 0.5 m then its speed when it makes an angle of 60° with the vertical is

(1) 2 m/s (2) 1 m/s (3) 4 m/s (4) 3 m/s

7. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. What is the linear speed of the motion

(1) 2.3 cm/s (2) 5.3 cm/s (3) 0.44 cm/s (4) none of these

8. The mass of the bob of a simple pendulum of length L is m. If the bob is left from its horizontal position then the speed of the bob and the tension in the thread in the lowest position of the bob will be respectively



(1) $\sqrt{2gL}$ and 3 mg (2) 3 mg and

(3) 2 mg and $\sqrt{2gL}$ (4) 2 gL and 3 mg

9. A stone of mass 1 kg is tied to the end of a string of 1 m length. It is whirled in a vertical circle. If the velocity of the stone at the top be 4 m/s. What is the tension in the string ?

(1) 6 N (2) 16 N (3) 5N (4) 10 N

10. If the speed and radius both are trippled for a body moving on a circular path, then the new centripetal force will be

(1) $F_2 = 2F_1$ (2) $F_2 = F_1$ (3) $F_2 = 3F_1$ (4) $F_2 = F/3$

11. The blades of an aeroplane propeller are rotating at the rate of 600 revolutions per minute. Its angular velocity is

(1) 10 π rad/s (2) 20 π rad/s (3) 2π rad/s (4) None of them

12. A particle moves in a circle of the radius 25 cm at two revolutions per second. The acceleration of the particle in m/sec^2 is

(1) π^2 (2) $8\pi^2$ (3) $4\pi^2$ (4) $2\pi^2$

13. When a body moves with a constant speed along a circle

(1) no acceleration is produced in the body (2) no force acts on the body

(3) its velocity remains constant (4) no work gets done on it

14. A particle moves in a circle describing equal angle in equal times, its velocity vector :–

(1) remains constant (2) changes in magnitude

(3) change in direction (4) changes in magnitude and direction

15. A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 r.p.m. Keeping the radius constant the tension in the string is doubled. The new speed is nearly
(1) 7 r.p.m. (2) 14 r.p.m. (3) 10 r.p.m. (4) 20 r.p.m.

16. In a vertical circle of radius (r), at what point in its path a particle may have tension equal to zero :–
(1) highest point (2) lowest point
(3) at any point (4) at a point horizontal from the centre of radius

17. If the radius of circular path of two particles of same masses are in the ratio of 1 : 2 and they have equal centripetal forces, then their velocities should be in the ratio of
(1) $1 : \sqrt{2}$ (2) $\sqrt{2} : 1$ (3) 4 : 1 (4) 1 : 4

18. A string of length 0.1 m cannot bear a tension more than 100N. It is tied to a body of mass 100g and rotated in a horizontal circle. The maximum angular velocity can be
(1) 100 rad/s (2) 1000 rad/s (3) 10000/s (4) 0.1 rad/s

19. The radius of the circular path of a particle is doubled but its frequency of rotation is kept constant. If the initial centripetal force be F, then the final value of centripetal force will be :–
(1) F (2) $\frac{F}{2}$ (3) 4F (4) 2F

20. A 0.5 kg ball moves in a circle of radius 0.4m at a speed of 4 ms^{-1}. The centripetal force on the ball is –
(1) 10N (2) 20N (3) 40N (4) 80N

21. A car is travelling at 20 m/s on a circular road of radius 100 m. It is increasing its speed at the rate of 3 m/s^2. Its acceleration is
(1) 3 m/s^2 (2) 4 m/s^2 (3) 5 m/s^2 (4) 7 ms^{-1}

22. A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 gm is tied to it and revolved in a circular path of radius 4m in a vertical plane. If g = 10 ms^{-2}, then the maximum angular velocity of the stone can be
(1) 16 rad/s (2) $\sqrt{21}$ rad/s (3) 2 rad/s (4) 4 rad/s

23. A stone attached to one end of a string is whirled in a vertical circle. The tension in the string is maximum when
(1) the string is horizontal
(2) the string is vertical with the stone at highest position
(3) the string is vertical with the stone at the lowest position
(4) the string makes an angle of 45° with the vertical

24. A weightless thread can withstand tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2m in a vertical plane. If g = 10 m/s^2, then the maximum angular velocity of the stone can be

(1) 5 rad/s (2) $\sqrt{30}$ rad/s (3) $\sqrt{60}$ rad/s (4) 10 rad/s

25. A particle moving along a circular path. The angular velocity, linear velocity, angular acceleration and certripetal acceleration of the particle at any instant respectively are $\vec{\omega}$, $\vec{v}$, $\vec{\alpha}$ are $\vec{a}_c$. Which of the following relation is/are correct

(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$ (c) $\vec{v} \perp \vec{a}_c$ (d) $\vec{\omega} \perp \vec{a}_c$

(1) a,b,d (2) b,c,d (3) a,b,c (4) a,c,d

26. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same then

(a) the centripetal force will not suffer any change in magnitude

(b) the centripetal force will have its direction reversed

(c) the centripetal force will not suffer any change in direction

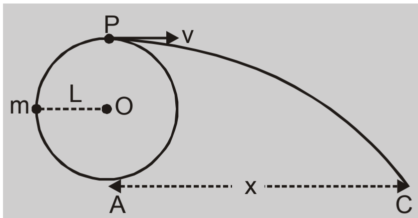
(d) the certripetal force is doubled

(1) a,b (2) b,c (3) c,d (4) a, c

27. A body tied to a string of length L is revolved in a vertical circle with minimum velocity, when the body reaches the upper most point the string breaks and the body moves under the influence of the gravitational field of earth along a parabolic path. The horizontal range AC of the body will be

(1) x = L (2) x = 2L

(3) x= $2\sqrt{2L}$ (4) x= $\sqrt{2L}$



28. A particle is moving in a vertical circle the tension in the string when passing through two position at angle 30° and 60° from vertical from lowest position are T_1 and T_2 respectively then

(1) $T_1 = T_2$ (2) $T_1 > T_2$ (3) $T_1 < T_2$ (4) $T_1 \geq T_2$

29. A body crosses the topmost point of a vertical circle with critical speed. What will be its centripetal acceleration when the string is horizontal

(1) g (2) 2g (3) 3g (4) 6g

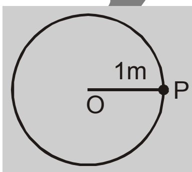
30. a_r and a_t represent radial and tangential acceleration. The motion of a particle will be uniform circular motion, if

(1) $a_r = 0$ and $a_t = 0$ (2) $a_r = 0$ but $a_t \neq 0$ (3) $a_r \neq 0$ but $a_t = 0$ (4) $a_r \neq 0$ and $a_t \neq 0$

31. Stone tied at one end of light string is whirled round a vertical circle. If the difference between the maximum and minimum tension experienced by the string wire is 2 kg wt, then the mass of the stone must be
(1) 1 kg (2) 6 kg (3) 1/3 kg (4) 2 kg

32. In uniform circular motion, the velocity vector and acceleration vector are
(1) Perpendicular to each other (2) Same direction
(3) Opposite direction (4) Not related to each other

33. A mass tied to a string moves in a vertical circle with a uniform speed of 5 m/s as shown. At the point P the string breaks. The mass will reach a height above P of nearly
(1) 1 m (2) 0.5 m
(3) 1.27 m (4) 1.25 m



34. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows, that
(1) its velocity is constant (2) its K.E. is constant
(3) its acceleration is constant (4) it moves in a straight line

35. If the overbridge is concave instead of being convex, then the thrust on the road at the lowest position will be
(1) $mg + \frac{mv^2}{r}$ (2) $mg - \frac{mv^2}{r}$ (3) $\frac{m^2v^2g}{r}$ (4) $\frac{v^2g}{r}$

36. A string of length 10 cm breaks if its tension exceeds 10 newtons. A stone of mass 250 g tied to this string, is rotated in a horizontal circle. The maximum angular velocity of rotation can be
(1) 20 rad / s (2) 40 rad / s (3) 100 rad / s (4) 200 rad / s

37. If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 s from its start is
(1) 8 rad/s (2) 12 rad/s (3) 24 rad/s (4) 36 rad/s

STD. XII

TIME :

Prof. SAMEER UNIA'S PHYSICS TUTORIALS

DATE :

PHYSICS

TOPIC : CIRCULAR MOTION - TUTORIAL SHEET - I

STUDENT NAME : ____________________

ANSWER KEY — EXERCISE - I

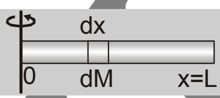
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	1	1	3	4	2	2	2	3	4	3	1	2	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	4	4	4	1	4	1	3	4	3	2	2	1	3	3
Que.	31	32	33	34											
Ans.	4	4	4	2											

1. $v = \omega r = 2\pi \quad 3 \quad 0.1 = 1.88$ m/s

$$a_c = \frac{v^2}{r} = \frac{(1.88)^2}{0.1} = 35.5 \text{ m/s}^2$$

$$T = \frac{mv^2}{r} = 1 \times 35.5 = 35.5\text{N}$$

3. Mass per unit length = λ



$dM = \lambda dx$

$dF = dM\, \omega^2 x$

$$F = \int_0^L \omega^2 x \lambda dx = \omega^2 \lambda \left[\frac{x^2}{2}\right]_0^L$$

$$= \frac{1}{2}\omega^2 \lambda L^2 = \frac{1}{2}\omega^2 (\lambda L)L \quad = \frac{1}{2}\omega^2 ML$$

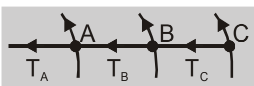
or

We can assume whole of the mass at CM which is at $\frac{L}{2}$ distance from axis of rotation

$$\therefore \quad F = M\omega^2 \frac{L}{2}$$

10. $F_C = \frac{mv^2}{r}$

$$F_C' = \frac{mv'^2}{r'} = \frac{m(2v)^2}{r} = 4F_C \qquad (r' = r)$$

11. 

angular velocity ω is same for all.

$T_C = m\omega^2 (3\ell)$

$T_B = T_C + m\omega^2 (2\ell) = m\omega^2 (5\ell)$

$T_A = T_B + m\omega^2 (\ell) = m\omega^2 (6\ell)$

$\therefore \quad T_C : T_B : T_A \quad :: 3 : 5 : 6$

14. $F = \frac{mv^2}{r} = \frac{500 \times (10)^2}{50} = 1000$ N

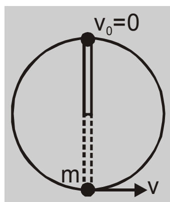
15. $T - mg = \frac{mv^2}{r} \quad \Rightarrow \quad 52 - 2 = \frac{0.2v^2}{0.1}$

$\Rightarrow \quad v = 5$ m/s

17. $\omega = \frac{2\pi}{60 \times 60} = \frac{\pi}{1800}$ rad/sec.

18. $a = \sqrt{a_T^2 + a_c^2} = \sqrt{(2)^2 + \frac{(30)^2}{500}} = 2.7$ m/s^2

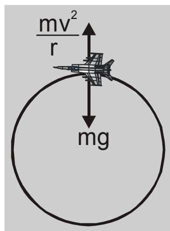
22. At top most point speed of the body may be zero, because rod will support the body their



$$\therefore \quad \frac{1}{2}mv^2 = 0 + mg(2\ell)$$

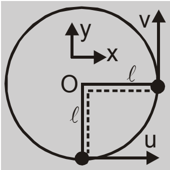
$$v = \sqrt{4g\ell}$$

23. $\frac{mv^2}{r} = mg$



$\Rightarrow \quad v = \sqrt{gr}$

24. $\vec{v} = v\tilde{j}$ and $\vec{u} = u\tilde{i}$



$$\frac{1}{2}mv^2 + mg\ell = \frac{1}{2}mu^2$$

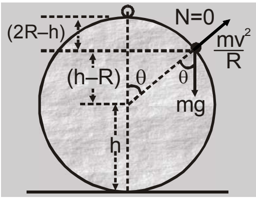
$\therefore \quad v = \sqrt{u^2 - 2g\ell}$

$\Delta\vec{v} = \vec{v} - \vec{u} = \sqrt{u^2 - 2g\ell}\,\tilde{j} - u\tilde{i}$

$|\Delta\vec{v}| = \sqrt{(\sqrt{u^2 - 2g\ell})^2 + u^2} = \sqrt{2(u^2 - g\ell)}$

25. When it will leave the sphere normal reaction become zero.

hence mg $\cos\theta = \dfrac{mv^2}{R}$



or $\quad \cos\theta = \dfrac{v^2}{Rg}$

$\dfrac{h - R}{R} = \dfrac{v^2}{Rg} \Rightarrow \quad v^2 = g(h - R)$

again $\dfrac{1}{2}mv^2 = mg(2R - h)$

$\dfrac{1}{2}g(h - R) = g(2R - h) \quad \Rightarrow h - R = 4R - 2h$

$3h = 5R \quad \Rightarrow \quad h = \dfrac{5}{3}R = \dfrac{5}{3} \times 21 = 35m$

26. $\omega = \dfrac{2\pi n}{T} = 2 \times \pi \times \dfrac{22}{44} = \pi$

centripetal acc. $a_C = \omega^2 r = \pi^2 \quad 1 = \pi^2\ m/s^2$

towards the center

27. Average velocity $= \dfrac{\text{Displacement}}{\text{Time taken}} = \dfrac{0}{62.8} = 0$

Average speed $= \dfrac{\text{Distance}}{\text{Time taken}} = \dfrac{2\pi r}{T}$

$= \dfrac{(2\pi)\,(100)}{(62.8)} = 10ms^{-1}$

29. At point B, $T - mg = \dfrac{mv_B^2}{\ell}$

By conservation of mechanical energy

$\dfrac{1}{2}mv_B^2 = mg\ell \Rightarrow T = mg + 2mg = 3mg$

30. $\because \dfrac{mv^2}{R} = mg \therefore v = \sqrt{Rg} = \sqrt{20 \times 10} = \sqrt{200}\ ms^{-1}$

32. Acceleration $= \omega^2 r = \left(\dfrac{2\pi}{T}\right)^2 r = \left(\dfrac{2\pi}{0.2\pi}\right)^2 \left(5 \times 10^{-2}\right)$

$= 5\ m/s^2$

STD. XII

Prof. SAMEER UNIA'S PHYSICS TUTORIALS

DATE :

TIME :

PHYSICS

TOPIC : CIRCULAR MOTION - TUTORIAL SHEET - II

STUDENT NAME : ____________________

ANSWER KEY — EXERCISE - II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	3	3	2	2	1	2	1	1	3	2	3	4	3	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	1	1	1	4	2	3	4	3	1	4	4	2	2	3	3
Que.	31	32	33	34	35	36	37								
Ans.	3	1	4	2	1	1	3								

1. $a_C = \frac{v^2}{r} = \frac{4}{r^2} \Rightarrow v^2 = \frac{4}{r}$

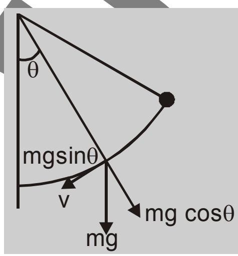
$\Rightarrow \quad v = \frac{2}{\sqrt{r}}$

Momentum $p = mv = \frac{2m}{\sqrt{r}}$

2. Uniform motion $a_t = 0$ and $a_c = \omega^2 r = v\omega$

Total acceleration $a_T = a_C = v\omega$

3. Tension at θ displacement



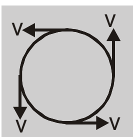
$T = mg\cos\theta + \frac{mv^2}{\ell}$

at extrem position v = 0

$\therefore \quad T = mg\cos\theta$

(at extreme position)

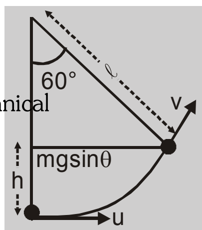
5. $\omega = \frac{\theta}{t} = \text{const.}$



– Uniform motion

– Magnitude of velocity is constant but the direction of velocity change

6. $h = \ell' - \ell\cos 60 = \ell - \ell \ \frac{1}{2}$



by law of conservation of mechanical energy

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$

$\therefore \quad v^2 = u^2 - 2g\frac{\ell}{2}$

$v = \sqrt{(3)^2 - 10 \times \frac{1}{2}} = 2$ m/s

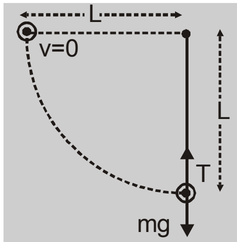
7. r = 12 cm ; $n = \frac{7}{100} = 0.07$ rev./sec

$\because \quad \omega = 2\pi n \quad \therefore \quad v = \omega r = 2\pi nr$

$v = 2\ \frac{22}{7}\ \frac{7}{100}\ 12 = 5.28$ cm/sec.

$\simeq 5.3$ cm/sec.

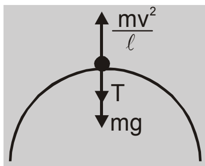
8. $0 + mgh = \frac{1}{2}mv^2 + 0$



$v^2 = 2gL \Rightarrow v = \sqrt{2gL}$

$T = mg + \frac{mv^2}{L} = mg + \frac{m}{L} \times 2gL = 3mg$

9. $T = \frac{mv^2}{\ell} - mg$



$= \frac{1 \times (4)^2}{1} - 1 \times 10 = 6$ N

10. $F_1 = \frac{mv^2}{r}$, $F_2 = \frac{m(3v)^2}{(3r)} = \frac{9mv^2}{3r} = 3F_1$

11. $n = \frac{600}{60} = 10\frac{\text{rev}}{\text{sec}}$

$\Rightarrow \quad \omega = 2\pi n = 2\pi \quad 10 = 20\pi$ rad/sec.

12. $\omega = 2\pi\, n = 2\pi \quad 2 = 4\pi$ rad/sec

$\because \quad a_t = 0$ (because ω = const.)

$\therefore \quad a_{net} = a_C = \omega^2 r = 16\pi^2 \quad 0.25 = 4\,\pi^2$

15. $T = m\omega^2 r$,

for constant r $\dfrac{T_2}{T_1} = \dfrac{\omega_2^2}{\omega_1^2}$

$\dfrac{2T}{T} = \dfrac{\omega_2^2}{(5)^2} \qquad \Rightarrow \qquad \omega_2^2 = 25 \quad 2$

$\Rightarrow \qquad \omega_2 \simeq 7$

17. $F_{C_1} = F_{C_2} \quad \Rightarrow \quad m\dfrac{v_1^2}{r_1} = m\dfrac{v_2^2}{r_2}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{r_1}{r_2}} = \dfrac{1}{\sqrt{2}}$

18. $T = m\omega^2 r$

$\dfrac{v_1}{v_2} = \sqrt{\dfrac{r_1}{r_2}} = \dfrac{1}{\sqrt{2}}$ = 100 rad/sec

19. $F_C = m\omega^2 r = m\,(2\pi\, n)^2 r$

$\because$ Angular frequency n = constant

$\therefore \quad F_C \propto r \qquad$ so $\qquad \dfrac{F_{C_2}}{F} = \dfrac{2r}{r}$

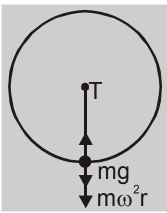
$\Rightarrow \qquad F_{C_2} = 2F$

20. $F_C = \dfrac{mv^2}{r} = \dfrac{0.5\times 4\times 4}{0.4} = 20N$

21. $a_t = 3\ m/s^2$ and $a_C = \dfrac{v^2}{r} = \dfrac{(20)^2}{100} = 4\ m/s^2$

$\Rightarrow \quad a_{net} = \sqrt{a_t^2 + a_c^2} = \sqrt{16+9} = 5\ m/s^2$

22. $T = mg + m\omega^2 r$



$\omega = \sqrt{\dfrac{T - mg}{mr}}$

$= \sqrt{\dfrac{3.7\times 10 - 0.5\times 10}{0.5\times 4}} = 4$rad/sec

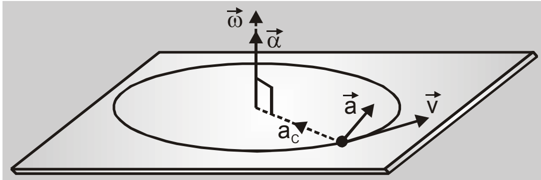
23. $T = mg + \dfrac{mv^2}{r}$

$\because$ v maximum at lowest point

$\therefore$ T maximum at lowest point

24. $\omega = \sqrt{\dfrac{T-mg}{mr}} = \sqrt{\dfrac{30 - 0.5\times 10}{0.5\times 2}} = 5$rad/sec

25.



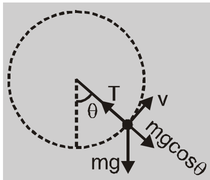
27. For looping the loop minimum velocity at top point $v = \sqrt{gL}$

time taken by particle

$t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 2L}{g}} = 2\sqrt{\dfrac{L}{g}}$

$\therefore$ horizontal range $x = vt = \sqrt{gL} \times 2\sqrt{\dfrac{L}{g}} = 2L$

28. $T = \dfrac{mv^2}{r} + mg\cos\theta$

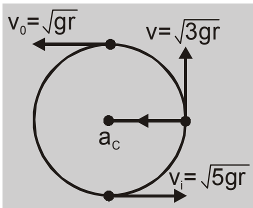


as θ increases cosθ and v both decreases

hence for θ = 60 , T will be less i.e., $T_1 > T_2$

29. $a_C = \dfrac{v^2}{r} = \dfrac{3gr}{r}$

$a_C = 3g$



30. In uniform circular motion.

magnitude of v = constant

so $\quad a_t = 0$; $a_r \neq 0$

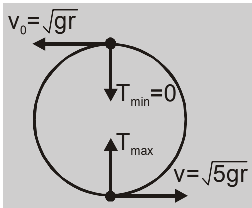
31. $T_{max} = mg + \dfrac{mv^2}{r} = mg + \dfrac{m}{r}\times 5gr = 6mg$

$T_{min} = 0$

$T_{max} - T_{min} = 6mg$

$\therefore \quad 6mg = 2g$

$m = \dfrac{1}{3}$kg



35. $N = mg + \dfrac{mv^2}{r}$